CHAPTER 11

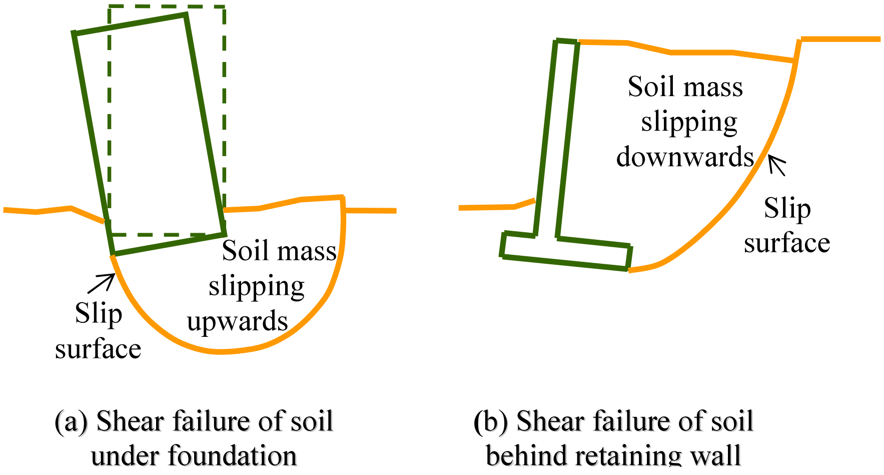
SHEAR STRENGTH OF SOIL

عندما تتعرض التربة للاجهادات تتحرك حبيباتها طبقا لطبيعة هذه الاجهادات واتجاهاتها وخواص التربة , وأثناء ذلك يحدث تغير فى محتوى المياه أو ضغط المياه بالتربة. وتكون كتلة التربة فى حالة اتزان طالما كانت المقاومة المتولدة بين حبيباتها أعلى من الاجهادات الواقعة عليها. فاذا زادت الإجهادات عن حدود معينة تستسلم التربة للحركة – بدون تغير ملموس فى الجهد – وتسمى هذه الحالة هندسيا بحالة الانهيار لكتلة التربة. وانهيار التربة يكون دائما فى صورة انزلاق لحبيباتها بعضها على بعض ولذلك فهو انهيار بالقص. ويصاحب هذا الانهيار تغير ملحوظ لشكل الأرض – مثل فى حالة انهيار الميول الترابية – او فى حالة انهيار التربة أسفل الأساسات مما يؤدى الى انهيار المنشأ.

11.1 Shear failure of soil

When a soil is subjected to vertical and lateral stresses, deformations in different directions may occur. The pore water pressure and water content of the soil may also change. When the stress exceeds a certain limit, accompanied by a certain strain, failure of the soil occurs. This failure is usually characterized by movement of the affected soil mass by slip (انزلاق) along a certain plane, which is called the **shear plane**.

For example in Fig. 11.1 (a), failure of the soil mass underlying the building foundation occurs due to excessive stressing of soil. The mode of failure (نمط الانهيار) indicates deformations and tilting of the building which finally results in development of the shown slip surface along which shear stresses exceed the shear strength of the soil (مقاومة القص للتربة).



(a) Shear failure of soil under foundation

(b) Shear failure of soil behind retaining wall

Fig 11.1 Soil failure by shear

Another example is given in Fig. 11.1 (b) where a slip surface develops due to excessive self weight of soil enhanced (معزز) by lateral displacement of the retaining wall which results from the high lateral pressure. Again, the shear stresses on the slip surface exceed the shear strength of the soil.

To judge on the safety of soil against shear failure (الانهيار القصى), the relations between stress and strain for the conditions under which the soil may fail have to be evaluated (تُقيّم).

11.2 The Mohr-Coulomb failure criterion

In a soil mass, if the shear stress along any plane passing through a point within the mass exceeds the shear strength of the soil, failure will occur. The failure shear strength (τ_f) of the soil on a failure plane or slip surface is expresses by the **Mohr–Coulomb** relationship which relates it to the normal stress (σ) acting on that plane as follows:

$$\tau_f = c + \sigma \tan\varphi \qquad \text{.. .. ..} (11.1)$$

(c) and (ϕ) are called the **shear strength parameters**, where (c) is the **apparent cohesion** and (ϕ) is the **angle of shear resistance**. Since the shear stresses in a soil mass can be resisted by the skeleton of the solid particles, the shear strength is expressed as a function of effective normal stress as follows:

$$\tau_f = c' + \sigma' \tan\varphi \qquad \text{.. .. ..} (11.2)$$

(c') and (ϕ') are called **effective shear strength parameters.**

In Fig. 11.2, if the shear and normal stresses on a plane within a soil mass are such that they plot at point (A), shear failure will not occur along that plane. However, shear failure will occur along a plane, if the stresses plot at point (B) which falls on the **failure envelop**. A state of stress given by point (C) will not exist.

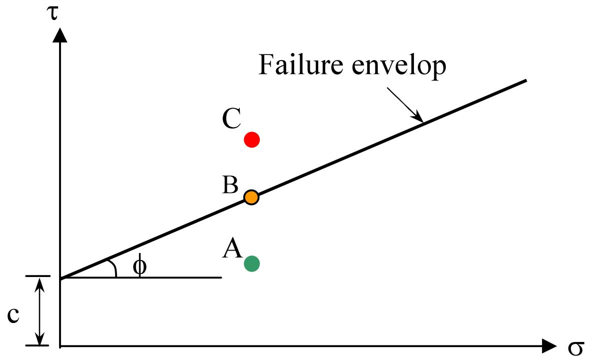


Fig 11.2 Mohr-Coulomb failure criterion

The shear strength of soil can be expresses in terms of the principle stresses at failure at a certain point (P) as follows:
This equation is represented graphically by **Mohr's circle** of stress, as shown in Fig. 11.3(a).

$$(\sigma_1 - \sigma_3) = (\sigma_1 + \sigma_3) \sin \varphi + 2c \cos \varphi \qquad \text{.. .. .. (11.3)}$$

If (σ_3) and (σ_1) are known, then the normal stress (σ) and the shear stress (τ) acting on the plane passing by point (P) and making an angle (α) with the plane on which (σ_1) acts can be calculated as follows, Fig. 11.3(b):

$$\sigma = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \,.\, \cos 2\alpha \qquad \text{.. .. .. (11.4)}$$

$$\tau = \frac{\sigma_1 - \sigma_3}{2} \,.\, \sin 2\alpha \qquad \text{.. .. .. (11.5)}$$

where: $\alpha = 45 + \frac{\varphi}{2}$ .. .. (11.6)

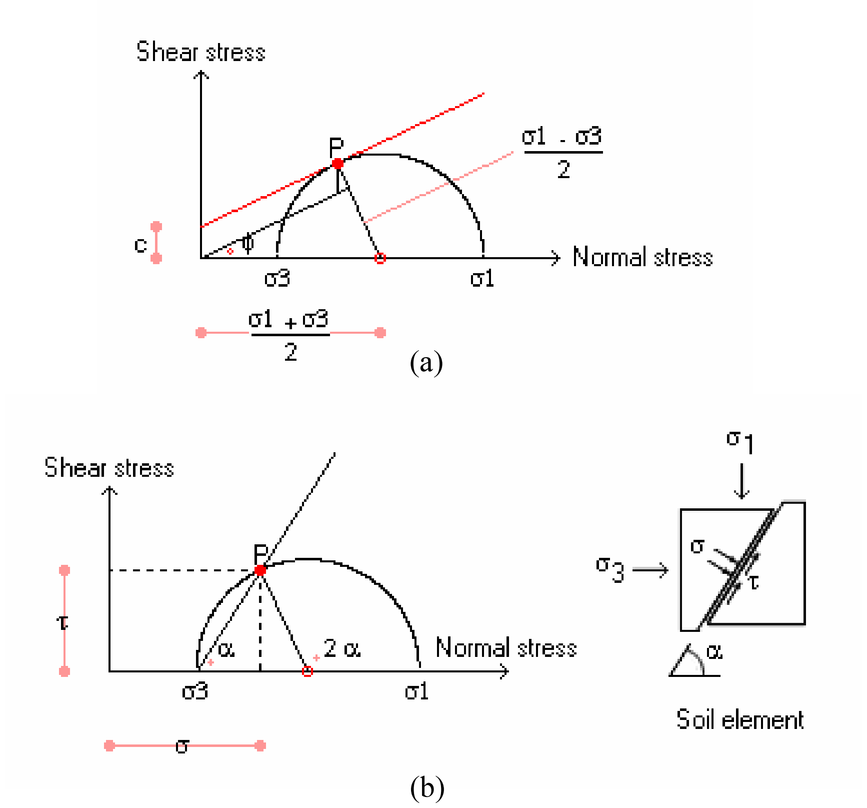


Fig 11.3 Mohr's circle presentation of stresses

The physical meaning of the Mohr–Coulomb relation is that the shearing resistance of soil consists basically of the following parameters:

(a) The frictional resistance to translocation (انتقال) between the individual soil particles at their points of contact.

(b) The structural resistance to displacement of the soil because of the interlocking between the particles.

(c) Cohesion, or adhesion, between the surfaces of soil particles.

The shear strength of cohesionless soil results from inter-granular friction alone, while in all other soils it results from both internal friction and cohesion.

Another method of presentation of the Mohr–Coulomb failure criterion is by plotting the **Modified failure envelop**, as shown in Fig. 11.4.

If points (P, P_1, P_2) lie on the modified failure envelope, the shear strength parameters of the soil (c', ϕ') can be obtained by measuring (a') and (α') and applying the following relations:

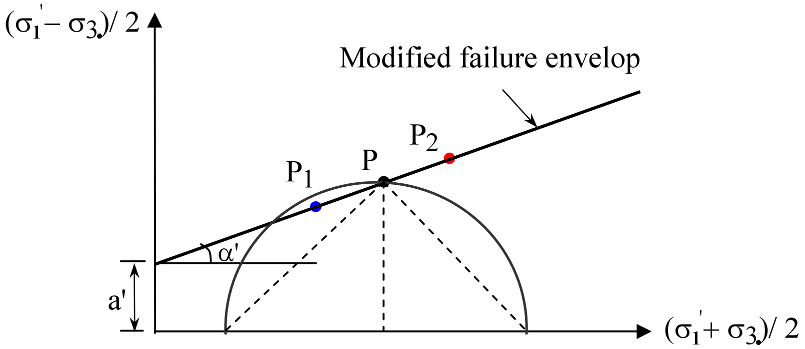


Fig 11.4 Presentation of stress condition by modified failure envelop method

$$\varphi' = \sin^{-1}(\tan\alpha') \qquad \text{.. .. .. (11.7)}$$

$$c' = a' / \cos\varphi' \qquad \text{.. .. .. (11.8)}$$

11.3 Behavior of granular soil in shear

Consider an assemblage (نسَق) of soil particles confined between two plates of rough surfaces. Let a vertical stress (σ) and shear stress (τ), both per unit gross area, be applied to the assemblage. The behavior upon application of (τ) depends on the looseness or denseness to which the grains are originally packed.

(a) **Loose state**:

When the vertical stress (σ) is applied, the distance between the plates decreases slightly, Fig 11.5. Then, the application of shear stress (τ) causes distortion associated with slipping between the grains, and a gradual rearrangement (اعادة تنظيم) of the particles into a denser packing, hence, the distance between the plates decreases more. Such decrease is greater than that which occurred due to (σ).

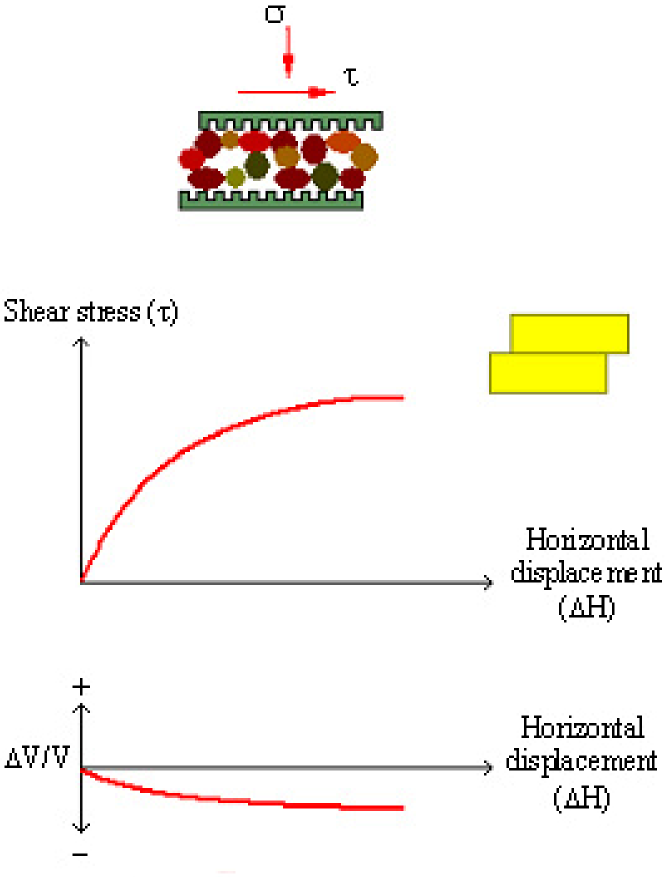


Fig 11.5 Distortion of granular soil in loose state

(b) **Dense state**:

When the vertical stress (σ) is applied, Fig. 11.6, the distance between the plates slightly decrease, lesser than in the case of loose state. Then, application of shear stress (t) forces causes the grains to move with respect of each other, but this cannot occur unless the distance (V), Fig. 11.6, between the plates increases. The shear stress at a given value of (σ) in the initial stage of test is much greater than that for the loose state. With the increase of distortion (تشوه), the particles spread (تنتشر بتباعد) to a density equal to that approached the loose packing at large distortions.

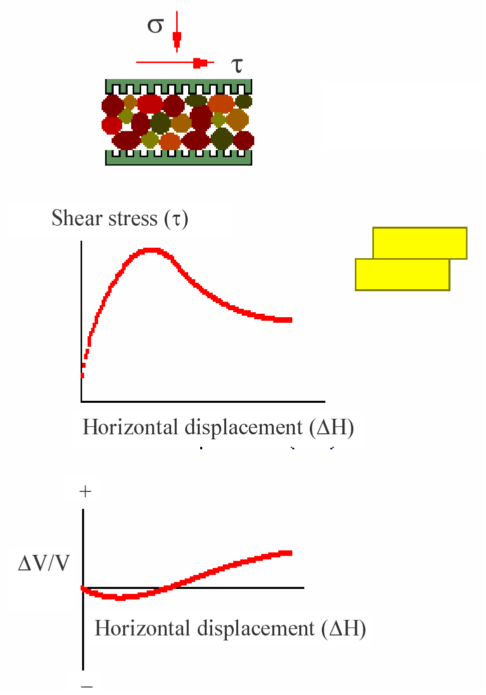


Fig 11.6 Distortion of granular soil in dense state

Therefore, with shear initial loose particles become denser, and initially dense particles become looser. This behavior is true for grains strong enough not to be crushed during shear. Sands and gravels closely approximate to this behavior.

11.3.1 Critical void ratio

Decrease of the volume of loose sand during shear is accompanied by decrease of void ratio, and increase of the volume of dense sand during shear is accompanied by increase of void ratio, Fig. 11.7. The void ratio for which no volume change occurs during shear is called the critical void ratio.

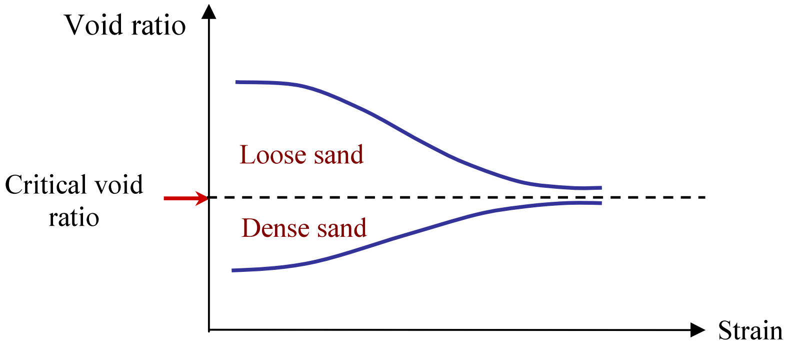


Fig 11.7 Definition of critical void ratio

11.4 Direct shear test

توضح العينة التربة فى صندوق القص المكون من جزئين وتوضح اعلاها احمال رأسية (عمودية على مستوى القص). يحرك احد نصفى الصندوق مسببا ذلك عينات متماثلة يمكن ايجاد العلاقة بين الجهد العمودى وجهد القص, ومنها يمكن تحديد معاملات مقاومة القص للتربة. يجرى الاختبار على عينات مقلقلة من التربة غير المتماسكة , كما يمكن اجراء الاختبار على عينات غير مقلقلة من التربة المتماسكة . وتجرى الاختبارات والعينات جافة او مبللة او مغمورة بالمياه و ولكن يلاحظ انه لايمكن التحكم فى مدى تصرف العينة او ضغط المياه البينية اثناء الاختبار.

The shear–deformation behavior of soils can be investigated by means of direct shear tests. The apparatus used in these tests is called the shear box. The set up and test apparatus are shown in Fig. 11.8.

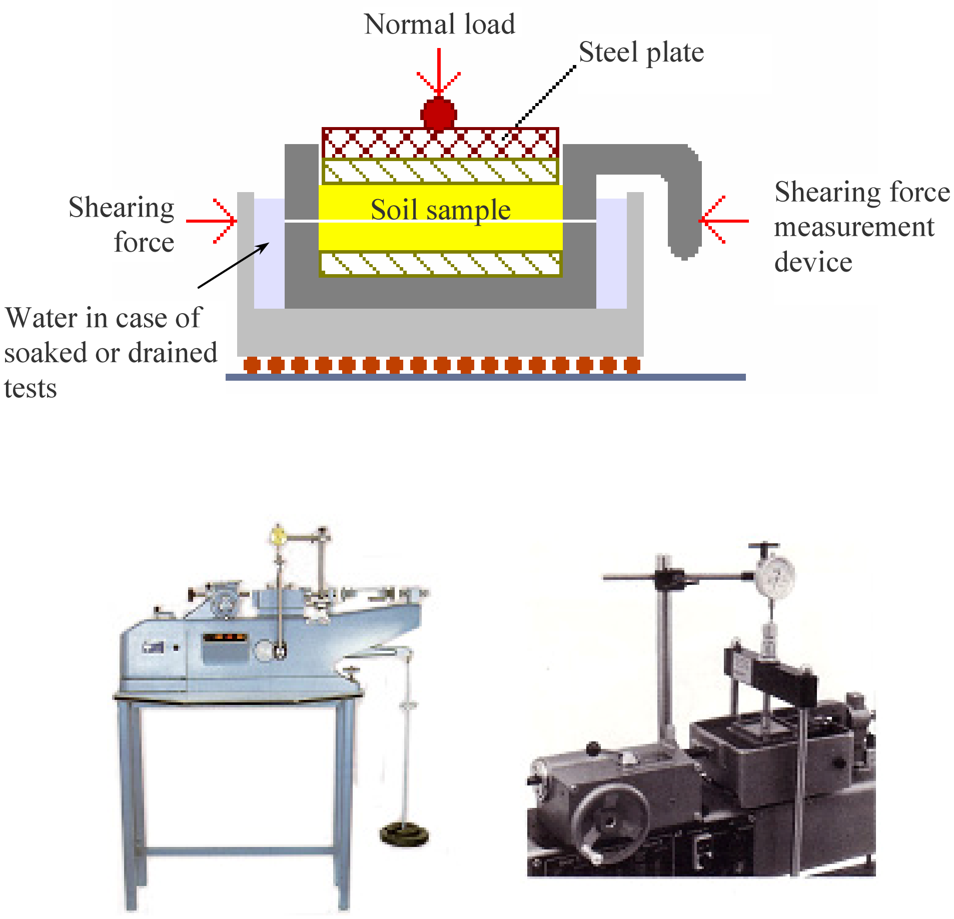

Normal load
Steel plate
Shearing force
Soil sample
Shearing force measurement device
Water in case of soaked or drained tests

Fig 11.8 Shear test box set up and apparatus

The application of both normal load and subsequent shearing force produces a change in the void ratio of the sample. If the sample is dry, or partially saturated, this change will occur very quickly. If the sample is saturated, the change may take time depending on the permeability of soil. The water content of the sample at failure depends on the degree of consolidation of the sample under the vertical load before shear, on the permeability of soil, on the rate at which shear force is increased, and no the drainage condition during test.

To investigate the effect of these factors, three types of test can be carried out:

(a) **Slow test:** the shearing forces is applied at a rate which will not permit pore water pressure to be built up due to shear, and the water content with the sample is always equalized (تتساوى).
(b) **Consolidated–quick test:** the sample is allowed first to consolidated under the vertical load, followed by quick shear at constant water content.
(c) **Quick test:** the water content of the sample remains unchanged during application of the vertical load and during the shearing stage (مرحلة القص)

In the shear box apparatus quick and consolidated quick tests can be carried out only on clayey soils. The permeability of other soils is so high such that even a very rapid increase of the stresses in the sample will cause some change in the water content. Slow tests can be carried out on any soil type.

In **loose sand** the shear stress increase with the increase of shear displacement, accompanied by decrease of volume, to a maximum, or ultimate, shear stress value (τ_{cv}) , at which the volume remains constant, Fig. 11.5. For **dense** and **medium dense sands** shear stress increase with the increase of shear deformation

up to a maximum peak value, approximately equal to (τ_{cv}) at large shear displacement attaining constant volume, Fig. 11.6.

The angle of friction of sand can be determined by plotting ultimate or peak shear stresses against corresponding normal stress, Fig. 11.9. Typical values of (ϕ) and (ϕ_{cv}) are given in Table 11.1. It is obvious that both values depend on soil density and shape of grains.

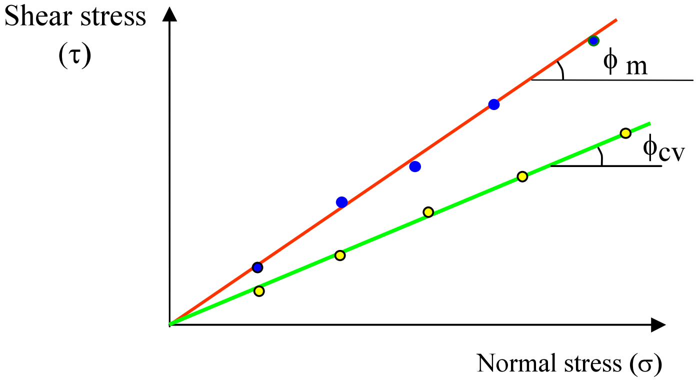


Fig 11.9 Determination of the angle of friction of sand

Table 11.1 Typical values of (ϕ) and (ϕ_{cv})
for granular soil

Type of soil	ϕ	ϕcv
Sand (round grains)		
Loose	28 – 30	
Medium	30 – 35	26 – 30
Dense	35 – 38	
Sand (angular grains)		
Loose	30 – 35	
Medium	35 – 40	30 – 35
Dense	40 – 45	
Sandy gravel	34 - 38	33 - 36

The shear box, however, has the following disadvantages:

(a) The area of shear surface changes as test progresses.
(b) Shear stresses are unequally distributed over the shear surface, Fig. 11.10.
(c) The water content of saturated samples of many types of soil changes as a result of change of stress.
(d) Shear failure does not take place simultaneously at every point of the shear surface. It starts at the edges and proceeds towards the centre. Therefore, the peak value of shear resistance measure in the test is lower than the real value.

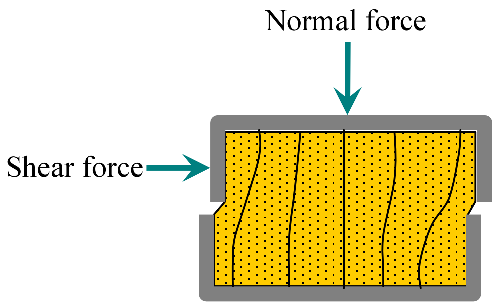


Fig 11.10 Unequal stress distribution in direct shear test

A modified from of the direct shear test is the simple shear test, Fig. 11.11 in which a rectangular specimen enclosed into a rubber membrane is placed inside a hinged box.

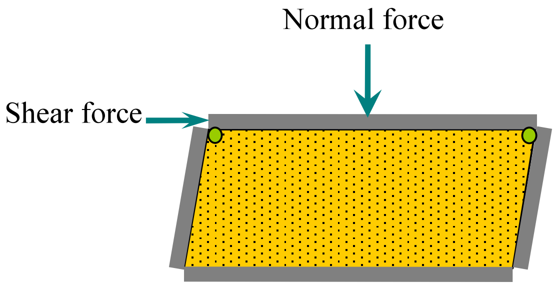


Fig 11.11 Simple shear test

11.4.1 Angle of repose

For loose sand and gravel the (σ - τ) relation is linear passing through the origin point, I.e. the intercept (c) is equal to zero. It can be represented accurately by the equation:

$$\tau = \sigma \tan \varphi_d \qquad \text{.. .. .. (11.9)}$$

where (ϕ_d) is called the angle of internal friction and equals roughly the angle of repose, Fig. 11.12.

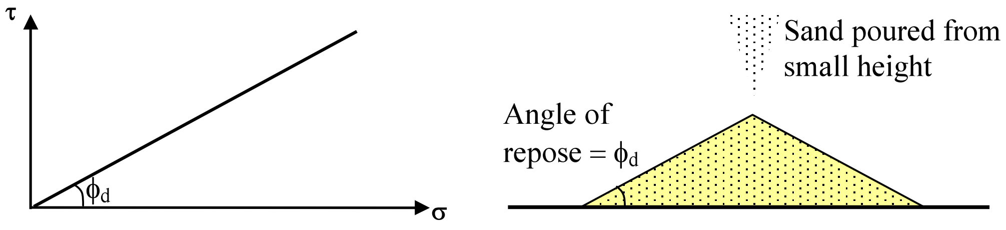


Fig. 11.12 Angle of internal friction of loose sand and gravel

11.5 Triaxial compression test

يتيح اختبار الضغط الثلاثى المحاور توقيع ضغوط جانبية ومحورية على عينة التربة, كما يتيح التحكم فى صرف المياه من العينة قبل وإثناء الاختبار. ويجرى الاختبار بأنماط مختلفة ليحاكى يقدرالامكان حالات الاجهادات فى التربة وبذلك تكون معاملات القص المستنتجة من الاختبار اقرب ما يمكن للواقع.

In the triaxial test, a cylindrical specimen is subjected to an equal all – round pressure and axial pressure. The test set up is shown in Fig. 11.13.

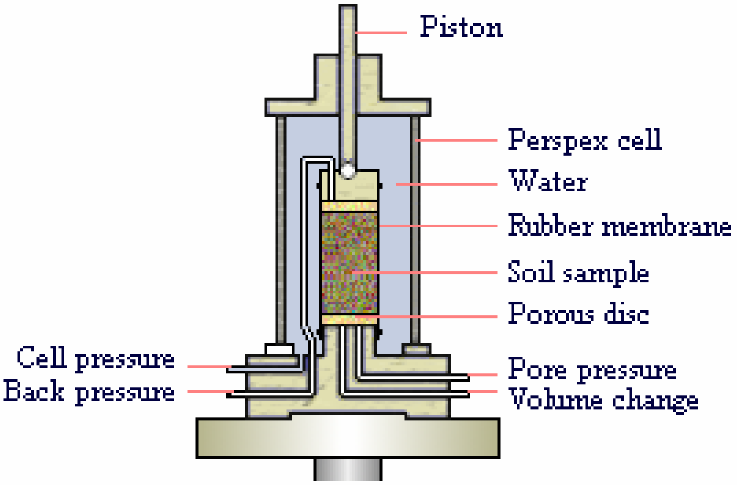


Fig. 11.13 Triaxial apparatus

The applied stress (σ_3) acts laterally on the cylindrical surface of the soil sample through the rubber membrane, and vertically through the top cap. The additional axial load is applied by means of a piston passing through the top of the chamber.

Figure 11.14 shows the used equipment which consists of both the triaxial cell and compression machine.

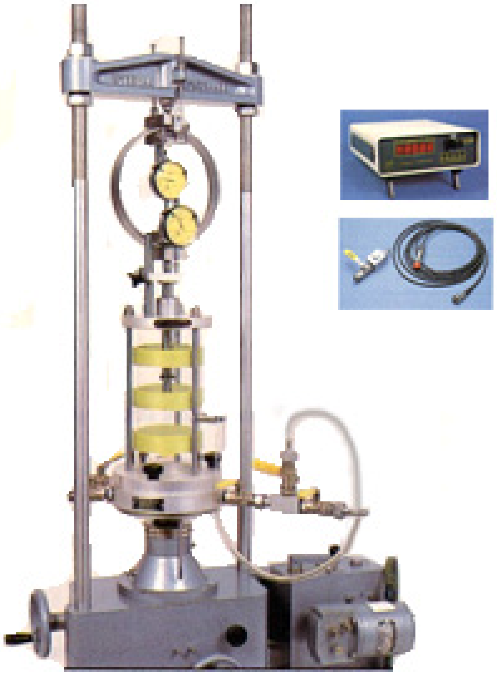


Fig 11.14 Triaxial compression machine and cell

By means of the connections through the cell the test can be done as

(a) Quick test (unconsolidated undrained test): No drainage is allowed.

(b) Consolidated–quick test: Drainage is allowed in the consolidation stage and volume of escaping water can be measure. Then, pressure in the water contained in the pores (pore water pressure) can be measure during the next stage of application of the vertical load.

(c) Drained (slow) test: Drainage is allowed in all stage of the test, and measurement of the quantity of drained water can be done.

As the loads are altered, the vertical deformation of the sample is measured by a dial gauge. This is usually carried out by applying (σ_3) then increase the axial stress, which is called "deviator stress" denoted (D), until failure occurs. Notice that:

$$D = \sigma_1 - \sigma_3 \qquad \text{.. .. ..} \ (11.10)$$

The stress – strain curve is usually plotted as ($\sigma_1 - \sigma_3$), or (D), against axial strain (ε_a), Fig. 11.15.

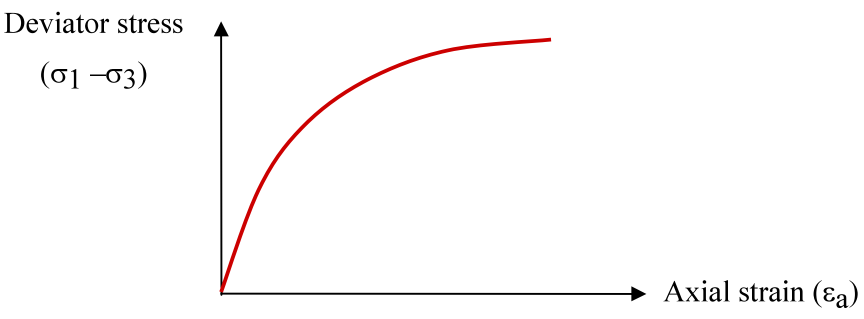


Fig. 11.15 Typical stress-strain relation for soil

11.5.1 Initial tangent modulus

The initial slope of the stress–strain relation, Fig. 11.16, is called the initial tangent modulus (E_i). It is used for calculating the "elastic" settlement of soil.

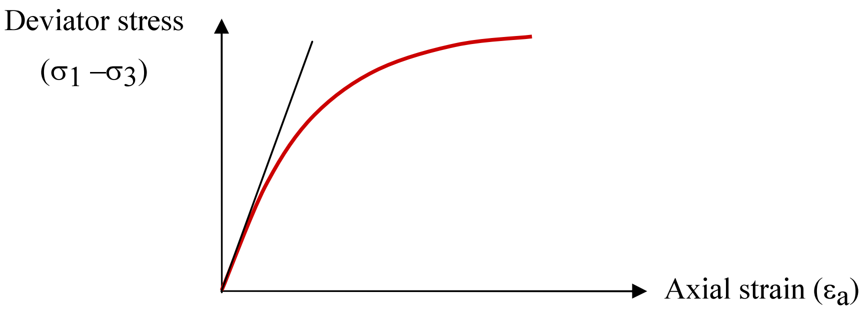


Fig. 11.16 Determination of the initial tangent modulus

11.6 Stress-strain relations for granular soil

In the test where the deviator stress (D) is done gradually and slowly, and drainage of water is allowed, no build up of pore water pressure occurs, i.e. This pressure remains zero and has no influence on the behaviour of the sample. A test conducted under this condition is called "**drained test**". This test is most accurate for determining the volume change curve by measuring the volume of water expelled from a loose sample, or taken into a dense sample.

Fig. 11.17 shows the results of drained triaxial tests on saturated sand. It is noticed that for loose sand (σ_1-σ_3) increase with the increase of (ε_a),

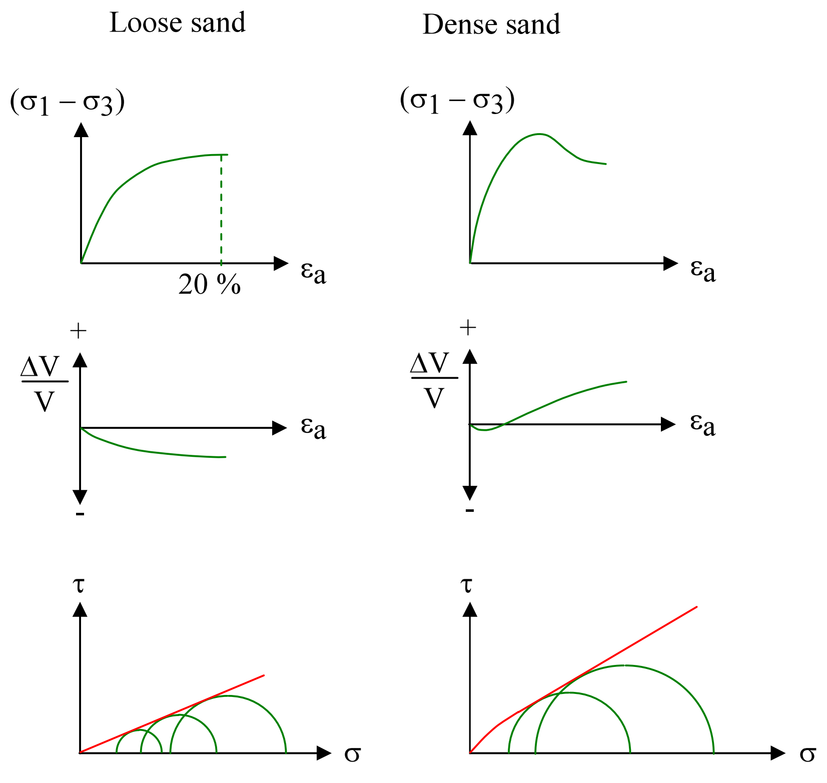


Fig. 11.17 Typical results of drained tests on saturated sand

approaching a limiting value designated "compressive strength". For convenience, this value is taken at 20% strain. The volume of sample decrease with the increase of (σ_1 - σ_3), and at final stages of test this increase approaches zero.

For dense sand (σ_1 - σ_3) increases with the axial strain to a peak value, then gradually reduces to the value approached by the loose sand. The compressive strength corresponds to the peak value. The volume initially decreases slightly, but as (σ_1 - σ_3) increases, it increases and the sample is said to "dilate" (تتمدد).

Typical **Mohr** diagrams for drained tests on saturated sand are also shown in Fig. 11.17. For dense sand the (σ - τ) relation is slightly curved downwards, but practically Equ. (11.9) applies. For gravels, sands and inorganic cohesionless silts (ϕ_d) depends on the relative density, grading characteristics and grain shape. Typical values are given in Table 11.2.

Table 11.2: Typical values of (ϕ_d)

Type of soil	ϕ_d (deg.)	
	Loose	Dense
Sand, rounded, uniform	27.5	27.5
Sand, angular, well graded	33	33
Sandy gravel	35	35
Silty sand	27 – 33	27 – 33
Inorganic silt	27 - 30	27 - 30

Comparing between the values given in Table 11.2 with those given in Table 11.1 it is noticed that the presence of water in the voids of sand and gravel has no influence on the (ϕ_d) values. But, if pressures develop in this water, effective stresses between the particles change, causing change of the shear strength and stress–strain relations. Development of pore water pressures depends on the drainage conditions, and on the tendency of the soil to dilate or contract.

اذا كانت خواص التربة وظروفها تقلل او تمنع صرف المياه البينية فانه سيتولد فى هذه المياه ضغط يعتمد على قابلية التربة للانضغاط او التمدد اثناء القص مما سيغير مقاومة القص التربة.

فاذا تعرضت تربة رملية مشبعة لاجهادات قص مفاجئة, مثل التى تحدث نتيجة زلزال او تفجير , وكانت نسبة الفراغات بها اعلى من نسبة الفراغات الحرجة (Critical void ratio) فنظرا لقابلية التربة للانضغاط اثناء القص وقصر زمن حدوث القص بما لا يتيح صرف المياه , فان ضغط المياه البينية سيرتفع . فاذا ارتفع ضغط المياه البينية ليصل الى قيمة الضغط الكلى اصبح الضغط الفعال مساويا للصفر وتنعدم مقاومة التربة . وتسمى هذه الظاهرة "تسييل" (Liquefaction) وتسبب انهيار كاملا للمنشات المقامة على التربة المفككة عند حدوث زلزال او تفجير.

A triaxial test can be carried out without allowing drainage of water after the sample is consolidated under all–round stress (σ_3). This type of test is called consolidated undrained test, termed (CU) test. Results of this test for sand are as shown in Fig. 11.18.

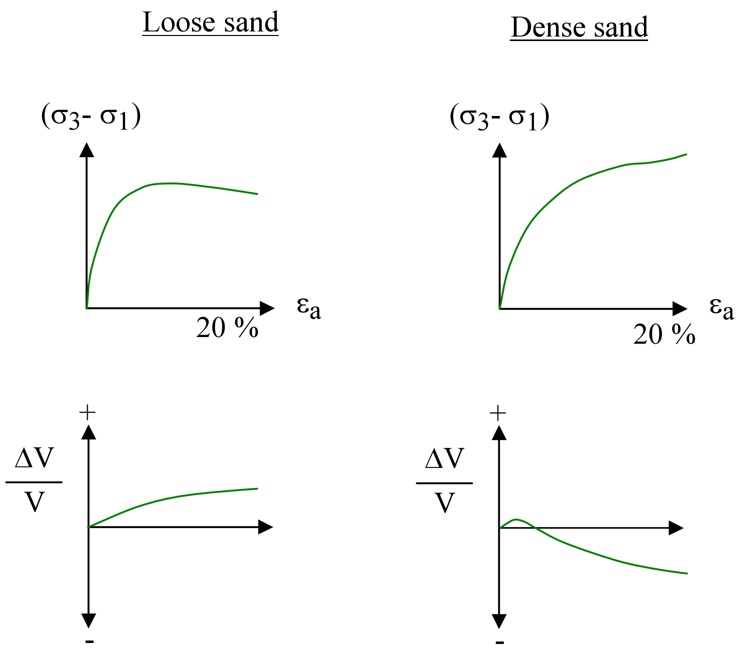
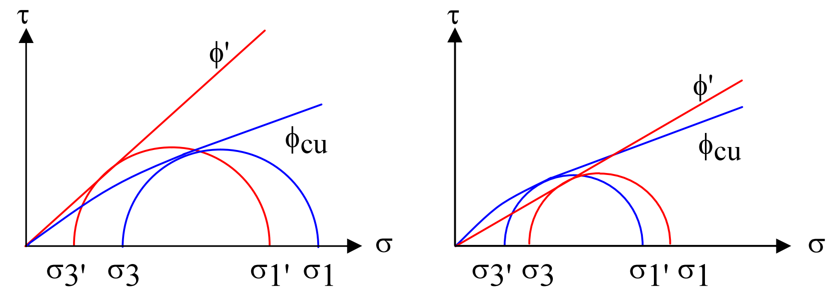


Fig 11.18 Typical results of consolidated undrained triaxial tests on sand

Notice that for loose sand, if the soil can not contract during the shear stage, due to earthquake or blasting, the pore water pressure (u) will increase. If the soil is very is very loose, (u) can approach (D), and liquefaction occurs. For dense sand the sample can be not dilate during shear, therefore, the pore water pressure decreases.

The values of (D) at failure are used to plot the Mohr circles such that the rupture lines are obtained. For loose sand, Fig. 11.19 (a), there is a little curvature. By measuring the pore water pressure at failure, Mohr circle corresponding to effective stress can be drawn..



(a) Loose saturated sand (b) Dense saturated sand

Fig 11.19 Mohr circles for rupture lines for (CU) triaxial tests on sand

It is found that the (ϕ') values are practically equal to (ϕ_d) obtained from the drained tests. For dense sand, if the circles corresponding to total stresses are drawn, Fig. 11.19(b), the envelop shows obvious curvature which increase for denser sand. The envelope for the effective stress circle is linear and (ϕ') is practically equal to (ϕ_d). Therefore, the (ϕ_d) value, called the **drained angle of internal friction** is a property for sandy soil independent on test conditions.

11.7 Shear strength of fine grained soil

The shear strength of saturated fine grained soil is dependent on the conditions of drainage before and during the shearing process. In the laboratory, the shear strength parameters can be mainly obtained for the following test conditions:

(a) Consolidated drained test (CD test)
(b) Consolidated undrained test (CU test)
(c) Unconsolidated drained test (UU test)

The **consolidated drained test** is carried out by applying (σ_3) in the triaxial cell on a sample and allow drainage of water due to consolidation of the sample; the volume of drained water is measured. Then, axial deviator stress is applied at a slow rate so that no pore water pressure is built up in the sample. The test is continued until failure occurs, either by drop of deviator stress, or if the axial strain reaches 20%. Typical relations between axial strain and deviator stress and Mohr diagrams are shown in Fig. 11.20.

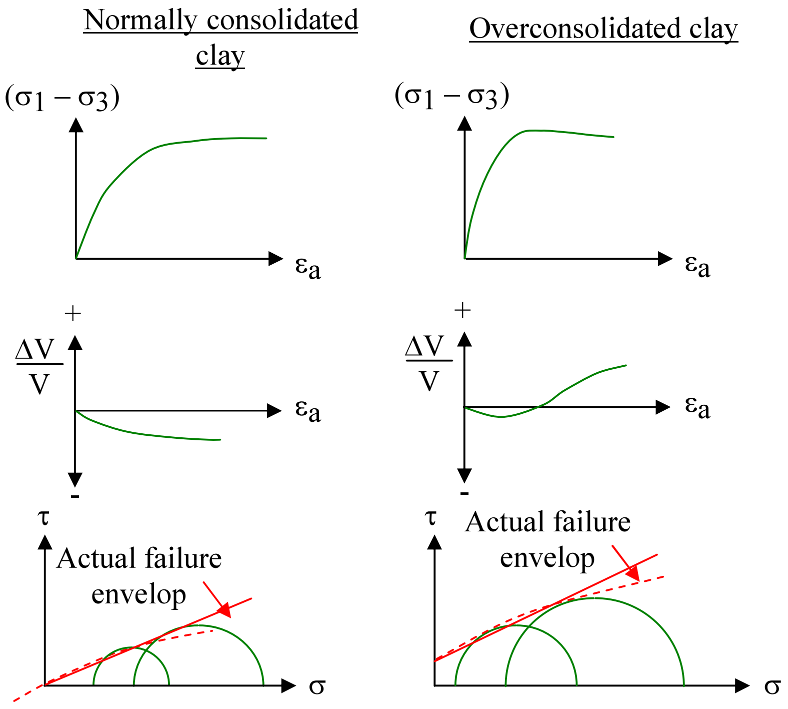


Fig. 11.20 Typical results of drained tests on clay

Failure of normally consolidated clays in drained tests occurs at relatively large strains and decrease of volume takes place during shearing. Over-consolidated clays fail at relatively small strains, and subsequently the deviator stress decreases with increase of strain. The fine grained soils at saturation are similar in behavior to sand. Normally consolidated clay behaves similar to loose sand, and over-consolidated clay behaves similar to dense sand. In addition, over-consolidated clay may possess significant shearing strength even in the drained test at ($\sigma_3 = 0$). The failure envelops for (σ_3) values less than the preconsolidation stress may be expressed by:

$$\tau_f = c_1 + \sigma' \tan\varphi_d \qquad \text{.. .. .. (11.11)}$$

For a certain type of clay, (ϕ_d) is nearly constant, but (c_1) is dependent on the over-consolidation stress.

لوحظ ان مقاومة الطين للقص مع صرف المياه تتناقص مع زيادة الانفعال. ولكن لايمكن الحصول على مقاومة القص الادنى من اختبار الضغط الثلاثى لان الجهاز يتيح انفعال لدرجة محدودة (كذلك صندوق القص يتيح انفعال لدرجة محدودة ايضا), لذلك فان انسب وسيلة لايجاد المقاومة الادنى, او المقاومة المتبقية للطين هى استخدام جهاز القص الحلقى .

In the triaxial test the "ultimate" deviator stress which remains constant with large strain, named the **residual strength**, can not be reached.

To overcome limitations of strain, so that the residual strength of the clay can be obtained, the ring shear apparatus is adopted. Since the residual strength on stress history, remolded samples are used. The sample is placed in the apparatus, Fig. 11.21, under a selected normal stress, then one half of the apparatus is rotated relative to the other by applying torque.

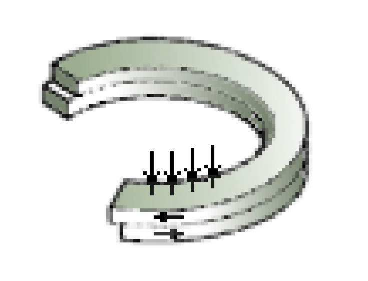


Fig. 11.21 Schematic drawing of the ring shear apparatus

There is no restriction (حظر) to the magnitude of the shear displacement along the shear plane. The rate of rotation is slow enough to ensure true drained condition. Shear stress is obtained by measuring the applied torque. The relation between shear displacement and shear is drawn under a selected normal stress, Fig. 11.22.

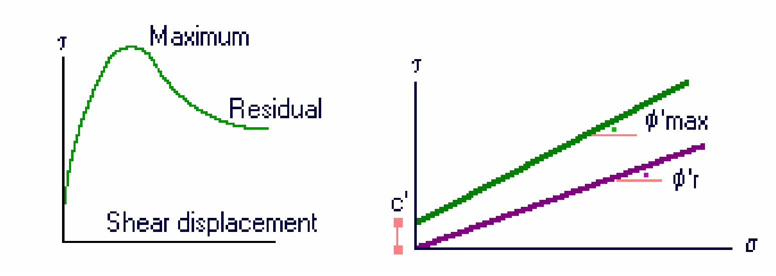


Fig 11.22 Residual strength of clay

The residual state is reached as the clay plat-like particles, which form the soil mass, are re-oriented (تغير من اتجاهها) parallel to the failure plane in a thin zone adjacent (قريبة جدا) to the failure plane. The residual angle of friction is obtained and used in long term analysis of slopes. Residual cohesion is normally zero.

The **consolidated undrained test** is carried out by applying (σ_3) in the triaxial cell on a sample and allow drained of water is measured. Then, the drainage outlets are closed and axial deviator stress is applied at a rate so that pore water pressure is equalized in the sample; measurement of pore water pressure is usually taken. The test is continued until failure occurs, either by drop of deviator stress, or if the axial strain reaches 20%. Typical relations between axial strain and deviator stress and Mohr diagrams are shown in Fig. 11.23.

For most foundation problems involving failure of fine grained soil, application of the Mohr–Coulomb relation for the drained condition is not possible. It is more convenient to apply results of tests in which the influence of pore pressure is taken into consideration. In many cases it is appropriate to investigate the behavior of soil under no pore pressure dissipation.

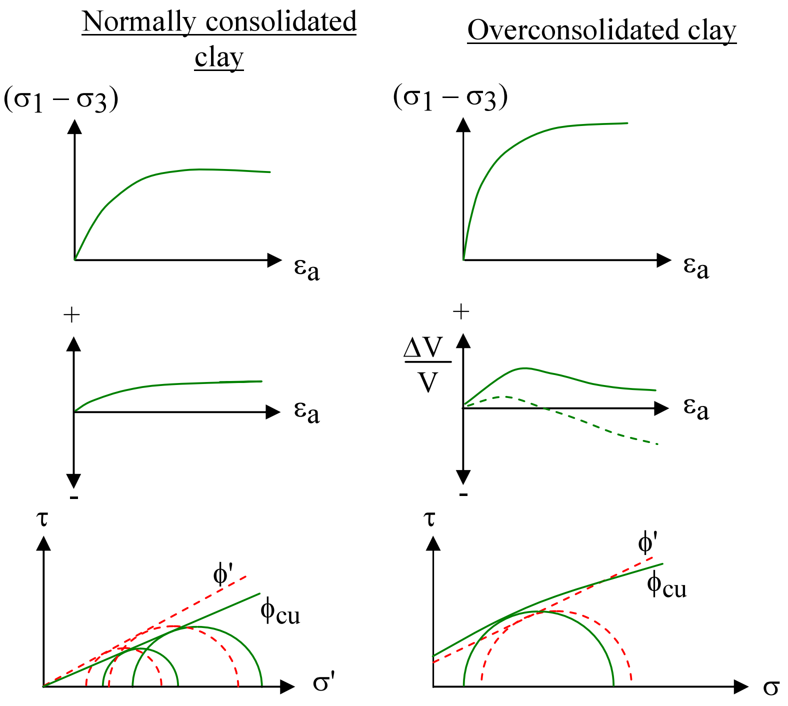


Fig. 11.23 Typical results of consolidated undrained triaxial tests on clay

The triaxial test can be carried out on a saturated soil sample without initial consolidation and without allowing water drained during loading. This is called the unconsolidated undrained test. Applying different confining pressure (σ_3) will cause different pore water pressure to develop within the sample; however, no change in the skeleton (structure) of the sample will occur.

If tests with different values of (σ_3) are carried out on identical samples (عينات متماثلة), the deviator stress at failure will be the same, Fig. 11.24. This is called the $\phi = 0$ condition.

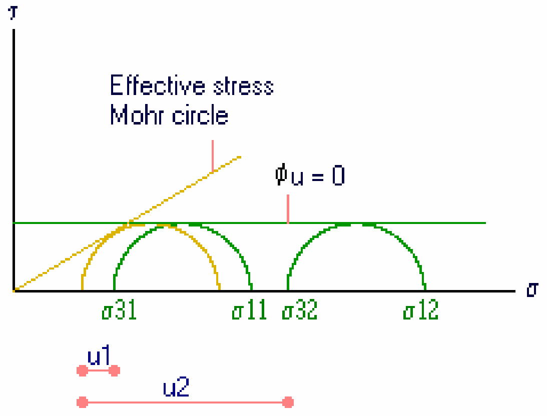


Fig. 11.24 Typical results of unconsolidated undrained triaxial tests on clay

Notice that since the skeleton of the samples is similar, all tests will lead to the same effective stress Mohr circle at failure. Again, if several samples are consolidated under the same cell pressure (σ_3), then tested under undrained conditions at different cell pressure, the rupture line with respect to total stress is horizontal intercepting the τ–axis at the undrained cohesion (c_u) and represented by the relation:

$$\tau = c_u = (\sigma_1 - \sigma_3)/2 \qquad \text{.. .. ..} (11.12)$$

11.7.1 Unconfined compressive strength test

The unconfined compressive strength test is carried out on cylindrical samples. Axial load is gradually increase till failure with no surrounding confining stresses applied. This test represents a special case of $\phi = 0$ condition where $\sigma_3 = 0$.

Any suitable compression machine can be used for carrying out the test. The autographic apparatus, Fig. 11.25, is most commonly used for this test using 38 mm diameter samples. This apparatus is portable which makes it convenient to be used for in – situ determination of the clay strengths. The test is simple and fast, however, there are limitations which include the following:

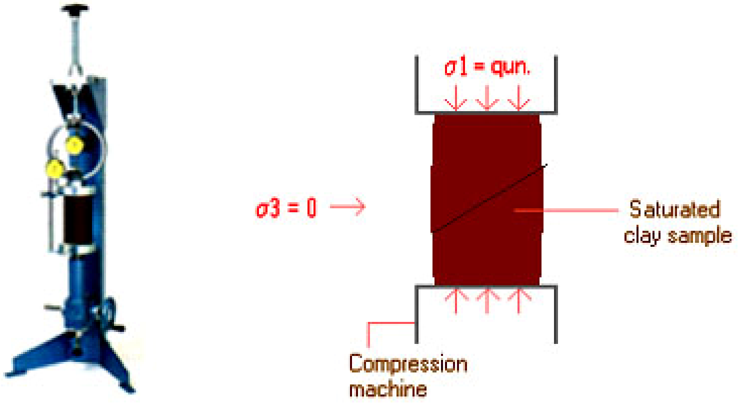


Fig. 11.25 Apparatus used for carrying out unconfined compressive strength test

(a) The tested sample must be fully saturated

(b) If the specimen contains any cracks or air voids, or has a low clay content, it will give untrue low strength.

(c) Since the drainage conditions are not controlled, the test must not be carried out too slowly, otherwise undrained condition will not be satisfied.

The Mohr circle representing the unconfined compressive strength test is as shown in Fig. 11.26. The shear strength of the soil (Cu) is given by:

$$c_u = q_{un.} / 2 \quad \text{.. .. .. (11.13)}$$

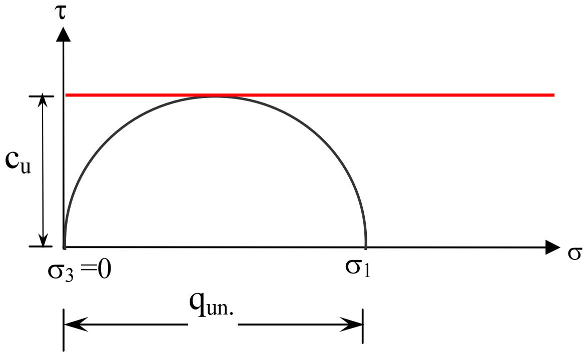


Fig 11.26 Mohr circle presentation for unconfined compressive strength test condition

يعتبر اختبار الضغط غير المحاط (او الضغط الحر) من اهم الاختبارات الروتينية التى تجرى على التربة الطينية المشبعة وتستخدم نتيجة هذا الاختبار فى تصنيف التربة الطينية وتدون نتائجه على قطاعات الجسات . ويمكن فى الحقل استخدام الجهاز المتنقل المبين ب (Fig. 11.25). كما يمكن فى الاعمال العادية استخدام جهاز الغرز الجيبى(Pocket penetrometer) المبين بـ (Fig. 11.27).

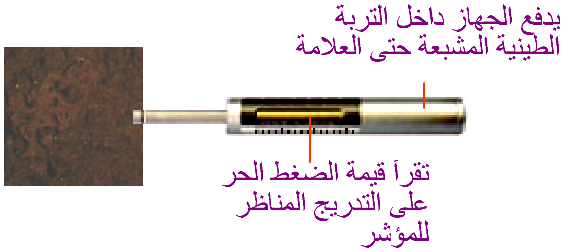

يدفع الجهاز داخل التربة الطينية المشبعة حتى العلامة
تقرأ قيمة الضغط الحر على التدريج المناظر للمؤشر

Fig 11.27 Pocket penterometer

Table 11.3 gives the classification of clayey soils consistency based on the unconfined compressive strength.

Table 11.3 Classification of clayey soil based on consistency

Consistency	$q_{un.}$ (kg/cm^2)	القـــــــوام
Very soft	< 0.25	لين جدا
Soft	0.25 – 0.5	لين
Firm	0.5 – 1.0	متوسط التماسك
Stiff	1 – 2	متماسك
Very stiff	2 – 4	شديد التماسك
Hard	> 4	جامد

11.7.2 Vane shear test

In this test a vane, Fig. 11.28 is rotated in the soil producing a direct shear test on mainly a cylindrical surface formed by the vane during rotation. The maximum torque applied is measured by a spring. No consolidation takes place before shearing, and the test is carried out quickly, so it is considered a (UU) test.

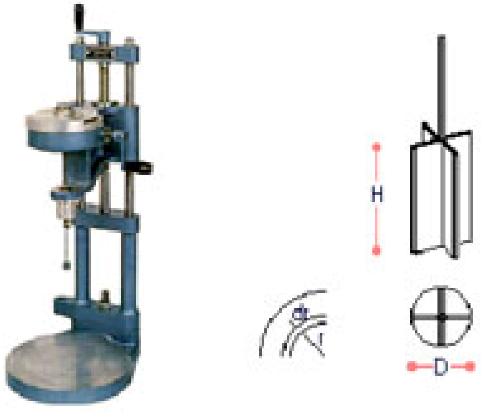


Fig 11.28 Shear vane apparatus

The test is suitable for saturated clays of very soft to firm consistency. Field vane tests are typically 50–75 mm diameter and 100 –150 mm long. Thin blades are used to decrease disturbance of soil due to penetration of blades into the soil.

Laboratory vanes, 13 mm diameter and 50 mm long, are used in the field in test pits (حفر اختبار).

The vane test overestimates the measured shear strength, depending on the plasticity of the tested clay. A correction factor is recommended to be used, Fig. 11.29. The field shear strength is calculated from the following relation:

$$c_u \ (\text{field}) = \mu \cdot c_u \ (\text{vane}) \qquad \text{.. .. .. (11.14)}$$

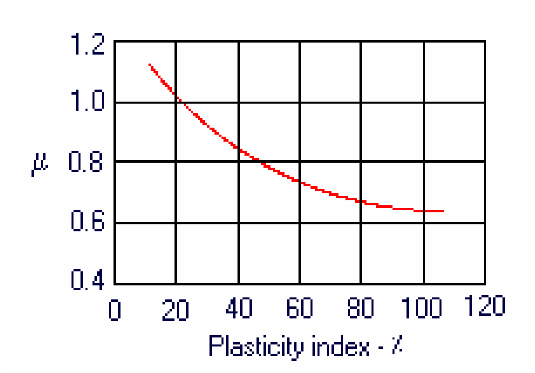


Fig 11.29 Correction factor for obtaining field shear strength of clay

To find (c_u) from the torque at failure obtained from the test, and dimensions of the vane:

$$T = (\pi \, . \, D \, . \, H \, . \, c) \, . \, (D/2) + 2 \int_{0}^{2D} (2\pi \, r \, . \, \Delta r \, . \, c) \, . \, r$$

$$T = \pi \ D^2 \ c \ (H/2 + D/6) \qquad \text{.. .. .. (11.15)}$$

11.7.3 Application of the (UU) test result

In case footing on clay, when the shear strength is determined by the (UU) test, it will be on the safe side, since some consolidation can take place. On the other hand, the clay beneath a deep excavation may tend to swell because of removal of the overburden stress. If the tendency to swell is more than that to consolidate due to positive pore water pressure induced by shearing stresses, the water content may increase, and the shear strength correspondingly decreases; the shear strength obtained from the (UU) test is not on the safe side of safety. The (ϕ = 0) concept for ove-consolidated clays generally leads to results on the unsafe side; (UU) tests can be applied where the over-consolidation ratio is 2 or may be 4.

It is also important to note that stiff saturated clays contain cracks, and the shearing strength of these deposits is to be determined using large diameter samples.

11.7.4 Relation between undrained cohesion and over-burden pressure for normally consolidated clays

For a normally consolidated clay, Fig. 11.30 shows that the ratio (Cu / σ_3') is constant. In a natural deposit, consolidation takes place under a combined (σ_1') and (σ_3') condition. However, it is found that there is constant ratio between (C_u), as determined from (UU) tests, and the overburden stress (σ_o'). Experiments proved that for normally consolidated clays, the (Cu/ σ_o') ratio depends on the plasticity index (P1- %), such that :

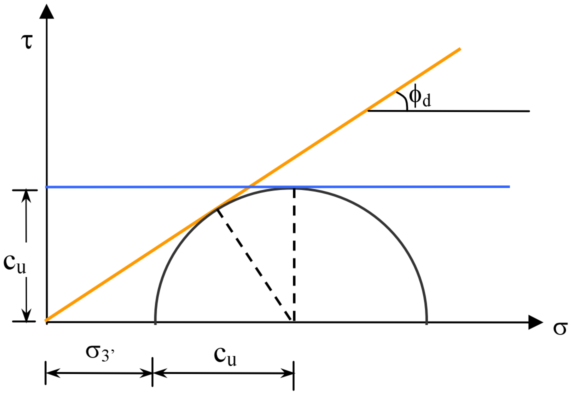


Fig 11.30 Mohr circle presentation of normally consolidated clay

$$c_u / \sigma_o' = 0.1 + 0.0037 \text{ P.I} \qquad \text{.. .. .. (11.16)}$$

11.8 Shearing resistance of unsaturated soils

The pore pressure in unsaturated soils is divided into pore water pressure and pore air pressure. These values may not be equal, and there is difficulty for their determination. In practice, tests on these soils are made to duplicate as closely as possible anticipated field conditions, using the triaxial test. In many case the (UU) test is appropriate. Although the water content is kept constant, volume change occurs due to compression of the air in the voids. Typical results on (CL) clay are shown in Fig. 11.31. It is noticed that curvature of the rupture lines decreases with the increase of the degree of saturation (S).

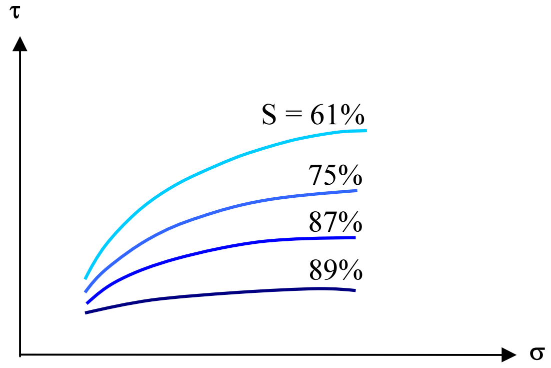


Fig 11.31 Rupture lines for unsaturated soil

For compacted clay fill, the deviator stress at failure, $(\sigma_1 - \sigma_3)_f$, decreases with the increase of placement water content (w). But, the fill can get saturated and the relationship between $(\sigma_1 - \sigma_3)$ and (w) can change as shown in Fig. 11.32.

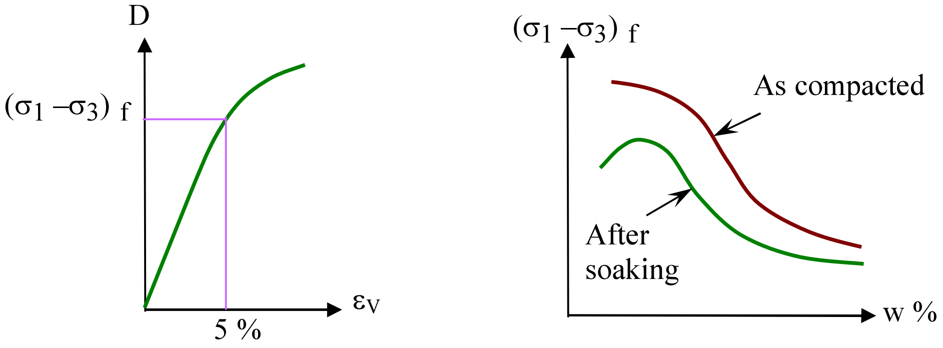


Fig 11.32 Typical relations for compacted soils

It may be required to test a compacted sample at placement water content after being soaked with water (as saturated samples) to obtain appropriate strength value.

11.9 Effect of rate of loading

Very slow rate of loading can occur in the field. Tests carried out at such rates showed that there is no effect for sands. Some clays, however, exhibit appreciable reduction of shear strength as the rate of loading decreases. This means that conventional (UU) tests may lead to unsafe estimate of the (c_u) field value. A reduction factor is suggested to be applied for the (c_u) laboratory value, or (c_u) field vane value for clays of high plasticity. The reduction factor (C_r) is given by:

$$C_r = 1 - 0.5 \log (PI / 20) \qquad \text{.. .. .. (11.17)}$$

11.10 Sensitivity and thixtropic characteristics of clays

Most undisturbed natural clays show reduction of strength when they are remolded. This characteristic of saturated cohesive soils generally called "sensitivity". It is defined by:

$$\text{Sensitivity} = \frac{q_{un.}(\text{undisturbed})}{q_{un.}(\text{remolded})} \qquad \text{.. .. .. (11.18)}$$

The classification of clays based on sensitivity is given in table 11.4.

Table 11.4 Classification of clayey soil based on

Senstivity	qun. – kg/cm^2	الحساسية
Insensitive	< 1	عديم الحساسية
Low sensitivity	1 – 2	قليل الحساسية
Medimum sensitivity	2 – 4	متوسط الحساسية
Sensitive	4 – 8	حساس
Extra sensitive	8 – 16	شديد الحساسية
quick	> 16	مفرط الحساسية

The sensitivity of most clays is generally in the range 1 to 8. However, highly flocculent marine clay deposits may have sensitivity higher than 16. Some clays turn to viscous liquids upon remolding; these clays are referred to as "quick clays". The loss of strength of clay soil from remolding is caused primarily by the destruction of the clay particle structure that was developed during the original process of sedimentation.

Thixotropy is defined as an isothermal, reversible, time-dependent process which occurs under constant composition and volume whereby the soil strength is reduced by remolding, then gradually returns to its original value when allowed to rest.

11.11 Selection of test procedures for determining the shear strength of soil in practice

Shear strength of soil depends on the pore pressures that exist at the time shear failure occurs. Excess pore pressure may be caused by the stresses to which the soil is subjected and by the volume change during shear. These pore pressures dissipate at a rate which depends on the soil permeability, on the soil mass influenced by shear stresses, and on the rate at which stresses are applied.

The following general remarks help in correct determination of the shear strength parameters of soils of different types:

(a) For sand and gravel with $k > 10^{-4}$ cm /sec, Equ. (11.2) can be applied with ($c = 0$) and ($\phi = \phi_d$). The value of (ϕ_d) can be determined from drained tests, or practically can be estimated from table 11.1. Estimates of the shear strength parameters can also be based on the results of conventional field tests such as the standard penetration or the cone penetration tests.

It has to be noted that if stresses are applied rapidly on a large soil mass of dense sand with permeability in the range 10^{-3} to 10^{-4} cm/sec, the value of (ϕ_d) obtained from slow tests can be conservative. But, loose sand in such conditions (ϕ') has to be obtained from (c_u) tests.

(b) For silts and clay the general procedure for determining the shear strength parameters is to use the triaxail test. Apply (σ_3') equal to the overburden pressure, then increase the deviator stress (D) until failure occurs. But, notice that for normally consolidated clays (σ_3') is less than the overburden pressure, thus the obtained shear strength parameters will be treated with higher factor of safety.

(c) For saturated soils with permeability less than 10^{-6} cm/sec, the triaxial (UU) test, unconfined test, or the vane test can be applied for determining the shear strength parameters. For fissured clay the (UU) triaxial test must be used.

(d) For long term analysis the shear strength parameters have to be determined from slow tests.

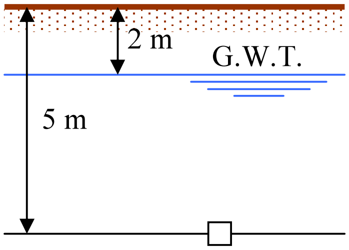
(e) For over consolidated clay (O.C.R > 6 and P.I. > 40) tests for determining the shear strength parameters are preferably carried out on large size samples.

(f) For partially saturated soils the shear strength parameters will be determined after saturating the sample.

11.12 Examples

(1) The soil at a site is formed of sand. The groundwater is 2 m below ground surface. The angle of internal friction of the sand is 35o, its dry density is 1.8 t/m3 and specific gravity 2.65. Find the shear resistance of the soil at a depth of 5 m. If the groundwater table rises to ground surface, find the change in the shear resistance.

Solution:



$$\gamma_d = \frac{G_s}{1+e}\gamma_w$$

$$1.8 = \frac{2.65}{1+e}\times 1$$

$$\therefore\ e = 0.472$$

$$\gamma_{sub.} = \frac{G_s - 1}{1+e}\gamma_w$$

$$= \frac{2.65-1}{1+0.472}\times 1 = 1.12\ \text{t/m}^3$$

For G.W.T. at depth 2 m :

$$\sigma_o' = 2\times 1.8 + 3\times 1.12 = 6.96\ \text{t/m}^2$$

$$\tau = \sigma\ \tan\phi = 6.96\times \tan(35) = 4.88\ \text{t/m}^2$$

For G.W.T. at greound surface :

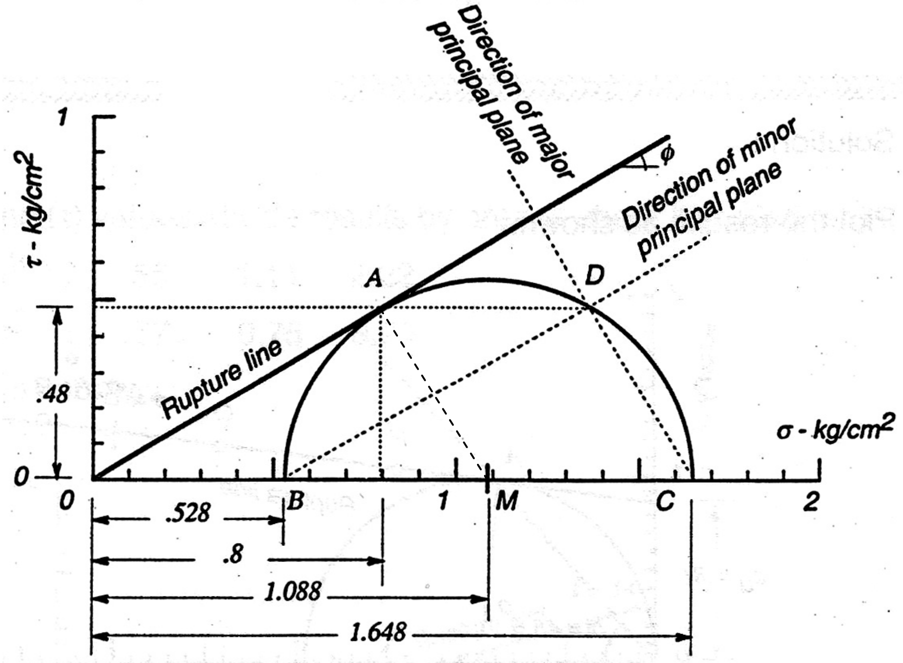
$$\sigma_o' = 5\times 1.12 = 5.6\ \text{t/m}^2$$

$$\tau = \sigma\ \tan\phi = 5.6\times \tan(35) = 3.92\ \text{t/m}^2$$

$$\therefore \Delta\tau = 4.88 - 3.92 = 0.95\ \text{t/m}^2$$

(2) A direct shear box test was carried out on dry coarse sand. The box is 6 x 6 cm. When a normal load of 28.8 kg was applied, the shear load at failure was 17.3 kg. find the angle of internal friction of the sand . Find the magnitude and direction of the principal stresses at failure.

Solution:



$$\sigma = \frac{28.8}{6 \times 6} = 0.8\,\text{kg/cm}^2$$

$$\tau = \frac{17.3}{6 \times 6} = 0.48\,\text{kg/cm}^2$$

Draw σ - τ plot : Point A = (0.8 , 0.48)

Join OA. This is the rupture line.

$$\phi = \tan^{-1}\frac{0.48}{0.8} = 31^{o}$$

From A draw AM $\perp$ OA. M is the centre of the Mohr circle.

Draw arc BAC.

$$\sigma_3 = OB = 0.528\,\text{kg/cm}^2$$

$$\sigma_1 = OC = 1.648\,\text{kg/cm}^2$$

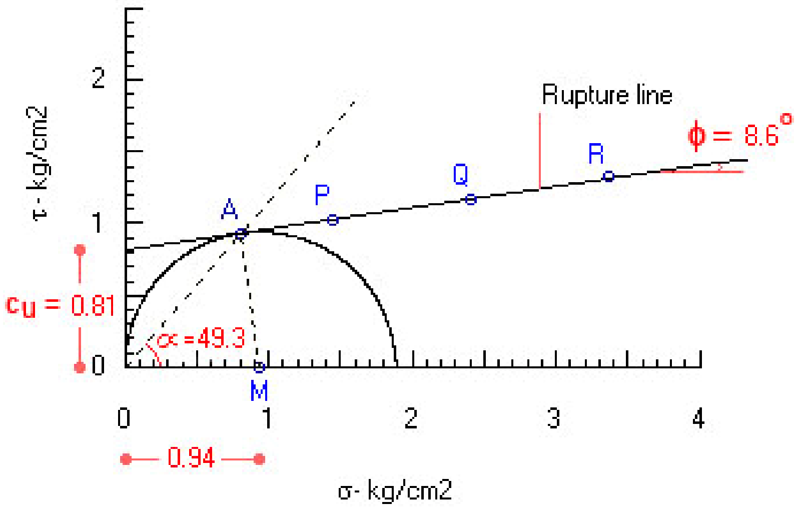
Direction of principal stresses at failure is obtained by drawing AD parallel to OC, joining DB and DC, as shown.

(3) Quick shear box tests were carried out on three specimens of partially saturated clay. Results are as follows:

Normal stress – kg/cm^2	:	1.45	2.41	3.37
Shear stress – kg/cm^2	:	1.03	1.17	1.32

Find the apparent cohesion and angle of internal friction. Determine the unconfined compressive strength of a specimen of the same soil.

Solution:



Solution:

Join the given points P, Q and R to obtain the rupture line, from which :

$c_u = 0.81\,\text{kg/cm}^2, \phi = 8.6^o$

For the unconfined condition $\sigma_3 = 0$.

To obtain the Mohr circle draw line OA making an angle α with the horizontal :

$$\alpha = 45 + \frac{\phi}{2} = 45 + \frac{8.6}{2} = 49.3^o$$

Draw AM $\perp$ rupture line. AMis the radius of the Mohr circle,

its value is equal to 0.94 kg/cm^2, which is c_u as will be obtained from the unconfined test.

$$\therefore q_{un.} = 2 \times 0.94 = 1.88\,\text{kg/cm}^2$$

(٤) Quick shear box tests were carried out on three specimens of sandy clay. The cross section of the shear box was 6 x 6 cm. Results are as follows:

Normal load – kg	:	20	40	80
Shear force at failure – kg	:	20.52	27	35.64

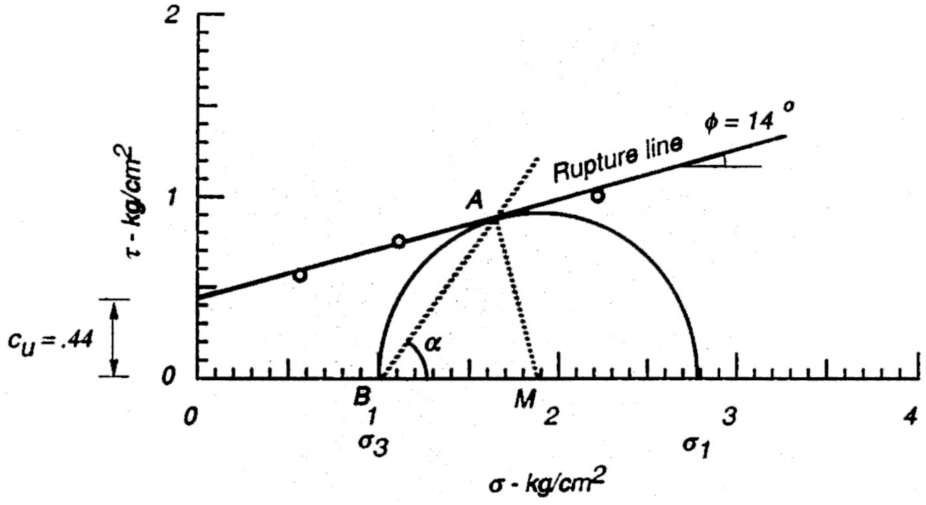
If a specimen of the same soil is tested in triaxial compression with cell pressure of 1 kg/cm^2, find the total axial stress at which failure will be expected to occur.

Solution:

To obtain σ and τ values, divide results by the area of the shear box (6 x 6 cm):

σ – kg/cm^2	:	0.56	1.11	2.22
τ - kg/cm^2	:	0.57	0.75	0.99

Plot the results as shown:



Draw the rupture line, from which $c_u = 0.44$ kg/cm^2 and $\phi_u = 14^o$.

To obtain the Mohr circle, plot $\sigma_3 = 1$ (point B),

calculate $\alpha = 45 + \frac{\phi}{2} = 45 + \frac{14}{2} = 52^o$.

Draw line BA making an angle α with the horizontal,

Draw AM $\perp$ rupture line. AM is the radius of the Mohr circle.

From the Mohr circle $\sigma_1 = 2.77$ kg/cm^2.

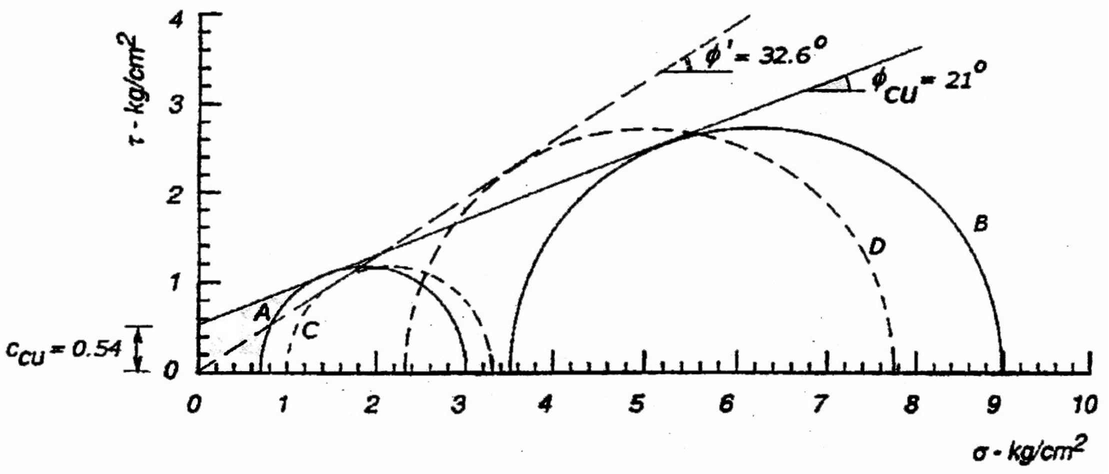
(5) The results of consolidated undrained triaxial tests with pore water pressure measurement on a compacted soil at failure are as follows:

Lateral pressure	kg/cm^2	:	0.7	3.5
Total vertical pressure	kg/cm^2	:	3.04	8.95
Pore water pressure	kg/cm^2	:	- 0.3	+ 1.17

Determine the apparent cohesion and angle of shearing resistance referred to total and effective stresses.

Solution:

To obtain c_{cu} and ϕ_{cu}, Mohr circle A and B are drawn.



From the common tangent: $c_{cu} = 0.54$ kg/cm^2 and $\phi_{cu} = 21^{o}$.

To obtain c' and ϕ', calculate σ_3' and σ_1' as follows:

$\sigma_3' = 0.7 - (-0.3) = 1.0,\ 3.5 - 1.17 = 2.23$ kg/cm^2

$\sigma_1' = 3.04 - (-0.3) = 3.34,\ 8.95 - 1.17 = 7.78$ kg/cm^2

Mohr circles C and D are drawn. From the common tangent:

c' = 0 and $\phi' = 32.6^{o}$.

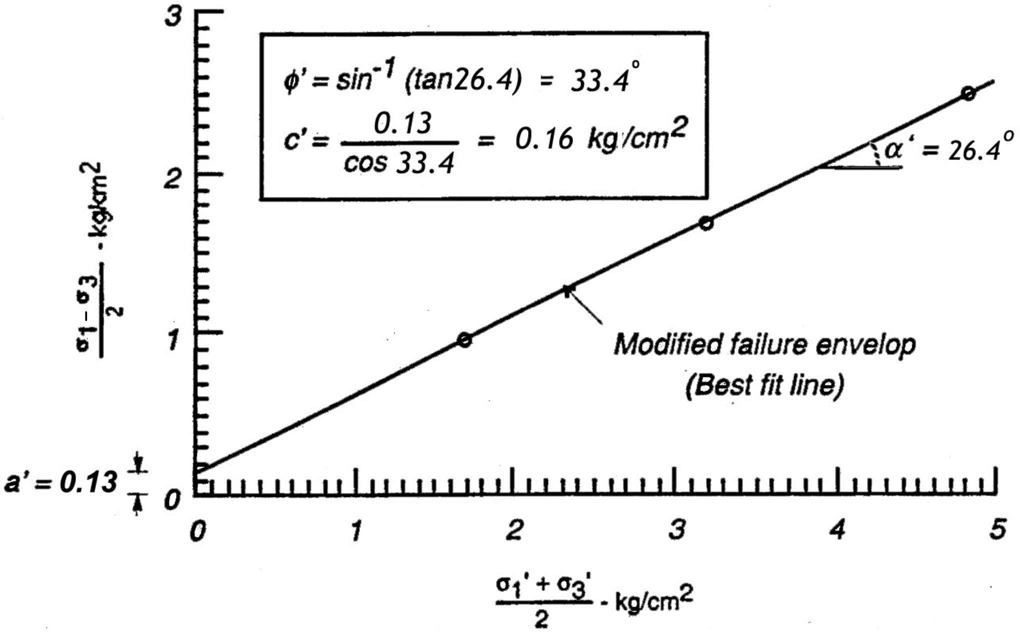
(6) The following results were obtained from consolidated undrained triaxial tests with pore water pressure measurement on saturated clay at failure. Determine the effective shear strength parameters (c' , ϕ'). Use the modified envelope method.

Cell pressure	kg/cm^2	: 1.50	3.0	4.5
Deviator stress at failure	kg/cm^2	: 1.92	3.41	5.04
Pore water pressure at failure	kg/cm^2	: 0.8	1.54	2.22

Solution:

The modified envelope is the relation between $\frac{\sigma_1'+\sigma_3'}{2}$ and $\frac{\sigma_1-\sigma_3}{2}$.

Test	Stress – kg/cm2					
	σ_3	σ_1	σ_3'	σ_1'	$\frac{\sigma_1'+\sigma_3'}{2}$	$\frac{\sigma_1-\sigma_3}{2}$
1	1.5	3.42	0.7	2.62	1.66	0.96
2	3	6.14	1.46	4.87	3.165	1.705
3	4.5	9.53	2.28	7.32	4.8	2.52



(7) A vane 10 cm long and 8 cm diameter was used to measure the shear strength of a soft clay. The torque at failure was 450 kg.cm. Further quick rotations causing remolding of the clay were applied and the required torque was 180 kg.cm. Calculate the undrained cohesion of the clay in its natural and remolded states.

Solution:

$T = \pi D^2 . c [H/2 + D/6]$

- Undisturbed state:

$450 = \pi (8)^2 . c_{u1} [10/2 + 8/6]$
$c_{u1} = 0.35 \text{ kg/cm}^2$

- Remolded state:

$180 = \pi (8)^2 . c_{u2} [10/2 + 8/6]$
$c_{u2} = 0.14 \text{ kg/cm}^2$

Senstivity = $q_{un.}$ (undisturbed)/ $q_{un.}$ (remolded)
= 2 x 0.35 / (2 x 0.14) = 2.5 (medium sensitivity)

11.13 Problems

(1) Specimens of sandy clay were tested in a shear box, 36 cm^2 in area, under undrained condition. The results are as follows:

Normal load	- kg :	18	36	54
Max. shear load	- kg :	11.52	15.84	20.16

Plot the failure envelope for the soil. Determine the apparent cohesion and apparent angle of internal friction.

(2) Specimens of compacted dry coarse sand were tested in a shear box, 6 x 6 cm. The results are as follows:

Normal load	- kg :	11	20	30	40
Peak shear load	- kg :	9.0	18.1	27	36.2
Ultimate shear load	- kg :	7.5	15.2	22.7	30

Determine the angle of shearing resistance in the dense and in the loose states.

(3) A specimen of dry cohesionless soil was tested in a shear box. At a normal stress of 2.4 kg/cm^2, the specimen failed at a shear stress of 1.6 kg/cm^2. Determine the angle of shearing resistance. Find from Mohr's diagram the magnitude and direction of the principal stresses at failure.

(4) Shear box quick tests were carried out on three specimens of unsaturated clay. The results are as follows:

Normal stress	- kg :	1.5	3	4.5
Shear stress at failure	- kg :	1.92	3.41	5.04

Find the apparent shear strength parameters. What will be the apparent cohesion obtained from the unconfined compressive strength test.

(5) Remolded specimens of soil, 6 cm square and 2 cm thick were tested in a shear box. Observations on the applied normal load and corresponding maximum shear load are as follows:

Normal Load	- kg :	18	27	36	45
Max. shear load	- kg :	15	19.5	25.5	31

Determine the shear strength parameters. What will be the value of the maximum shearing load, if an identical specimen of the soil is tested under normal load of 50 kg. Draw Mohr's circle for this applied load. Indicate the direction of the plane of failure, and the planes of major and minor stresses. Determine the magnitude of the principal stresses.

(6) A drained triaxial compression test was carried out on a saturated specimen of clean sand. When the cell pressure was 1 kg/cm^2, the deviator stress at failure was 4.82 kg/cm^2. Find (ϕ_d). Determine the deviator stress and major principal stress at failure, if the cell pressure was 2 kg/cm^2.

(7) Two specimens of unsaturated compacted soil were tested in the triaxial cell under undrained condition. Each specimen was 4 cm diameter and 8 cm high. Results of the tests are as follows :

Test No.		:	1	2
Cell pressure	- kg/cm^2	:	1	2
Deviator load	- kg	:	72	91.5
Volume change	- cm^3	:	+ 1.2	+ 1.6
Axial deformation	- cm^3	:	- 0.6	- 0.8

Determine the shear strength parameters.

(8) Triaxial tests were carried out on two specimens of soil. The results are as follows :

Cell pressure	- kg/cm^2	:	1	2
Deviator stress at failure	- kg/cm^2	:	3	6

Determine the shear strength parameters of this soil. If a third identical specimen was tested under a cell pressure of 3 kg/cm^2, find the values of the major and minor principal stresses.

(9) In an undrained triaxial test, a specimen of saturated clay failed under a deviator stress of 1.8 kg/cm^2 when the cell pressure was 0.75 kg/cm^2. Determine the value of the undrained shear strength of the soil. What will be the measured unconfined compressive strength of another specimen of the same clay. Draw Mohr's circles for both tests showing the failure envelopes.

(10) The following are results of consolidated undrained triaxial tests with pore water pressure measurements on two specimens of a soil :

σ_3 -	kg/cm^2 :	1	2
σ_1 -	kg/cm^2 :	4.4	7.6
u -	kg/cm^2 :	- 0.2	+ 0.6

Determine the total and effective shear strength parameters.

(11) Consolidated undrained triaxial tests with pore water pressure measurements were carried out on four specimens of silty clay. Results are as follows :

σ_3 -	kg/cm^2 :	0.5	1	1.5	2
σ_1 -	kg/cm^2 :	3.5	4.4	5.3	6.1
u -	kg/cm^2 :	0.05	0.1	0.12	0.18

Draw the modified failure envelope. Determine the effective shear strength parameters.

(12) A drained test was carried out on dense sand. The all round pressure was 0.75 kg/cm^2 and the added vertical pressure to cause failure was 2.715 kg/cm^2. Calculate the angle of internal friction and the angle of inclination of the failure plane.

(13) In a consolidated undrained triaxial test with pore water pressure measurement on dense sand, the sample was consolidated under all round pressure of 0.75 kg/cm^2. At failure the pore water pressure was 0.6 kg/cm^2 and the added vertical pressure was 0.725 kg/cm^2. Calculate the angle of internal friction.

(14) a- Derive the relation between the major and minor principal stresses at failure in terms of c and ϕ.
b- In a drained triaxial compression test, a saturated specimen of sand failed under a deviator stress of 5.35 kg/cm^2 when the cell pressure was 1.5 kg/cm^2. Find the effective angle of shearing resistance of the sand, and the approximate inclination of the failute plane to the horizontal. What will be the deviator stress and the major principal stress at failure for another identical specimen, if it is tested under a cell pressure of 2 kg/cm^2.

(15) Remolded specimens of silty sand prepared by compaction to Proctor maximum dry density and optimum moisture content are used for consolidated

undrained triaxial tests with pore water pressure measurements. The results are as follows:

Test No.	:	1	2	3	4
σ_3' - kg/cm^2	:	0.4	1	1.65	2.75
$\sigma_{1'}$ - σ_3' - kg/cm^2	:	3	4.43	6.15	8.8

Draw the Mohr circles and the failure envelopes. Determine the values of c' and ϕ'. Also plot the modified failure envelope and compare the results.

(16) A drained triaxial test is carried out on a uniform dense sand with rounded grains. .The all round pressure $\sigma_3 = 1$ kg/cm^2. Estimate the deviator stress at failure. Use Table 11.1 for typical ϕ_d value.

(17) The shear strength parameters of a soil were reported to be c = 0.2 kg/cm^2 and $\phi = 22^o$. What is the minimum lateral pressure required to prevent failure of the soil at a point subjected to vertical pressure of 4.5 kg/cm^2.

(18) Undrained triaxial test was carried out on a sample of saturated clay under a lateral pressure of 0.5 kg/cm^2. Calculate the deviator stress at failure knowing that the unconfined compressive strength is 1.81 kg/cm^2.

(19) The soil under a footing consists of coarse sand having and angle of shearing resistance of 35^o. The footing is subjected to an average vertical pressure of 1.2 kg/cm^2. An excavation was then carried out in the region rounding the footing. The footing settled and the underlying soil failed in shear. Estimate the shear stress acting on the failure plane.

(20) An unconfined compressive strength test was carried out on a specimen of clay 4 cm diameter and 8 cm high. The specimen failed under an axial load of 46.5 kg. At failure the axial change in the length of the specimen was 1 cm. Find the unconfined compressive strength of the clay. Describe the clay with respect to its consistency.

(21) An unconfined compressive strength test was carried on a soil sample. The axial stress at failure was 1.6 kg/cm^2. The plane of failure was inclined 50^o to the horizontal. Calculate the cohesion and angle of internal friction of the soil.

(22) Calculate the shearing resistance against sliding along a horizontal plane at a depth 7 m in a soil deposit consisting of coarse sand. The groundwater table is at a depth of 3 m. The bulk density of the sand above the groundwater table is 1.89 t/m^3, and the saturated density below groundwater table is 2.05 t/m^3. Assume that the angle of internal friction of the sand is $32\,^{o}$.

(23) A vane 11.4 cm long, 7.6 cm in diameter was pressed into soft clay at the bottom of a borehole. Torque was applied and gradually increased to 450 kg.cm when failure took place. Find the shear strength of the clay.

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